

# Determining the Coefficient of Thermal Expansion of a Piezoelectric Transducer using a Balanced Michelson Interferometer

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To design an optical cavity that is passively stabilized by precisely controlling its temperature, it is crucial to know thermal expansion coefficients of all relevant parts with high precision. The cavity mirrors and piezo transducer (PZT), have significant thermal expansion coefficients ( $\alpha > 1 \times 10^{-6}/\text{K}$ ). To reduce such possible thermal drift by a factor of ten and, to offset this thermal drift, a metal shim will be placed between the ultra-low expansion (ULE) glass cavity support and the cavity mirror assembly. A Michelson interferometer was implemented to measure a small spatial shift that is induced by the PZT thermal expansion. The coefficient of thermal expansion of the PZT was found to be  $-5.6 \pm 1.2 \frac{\mu\text{m}}{\text{m}^\circ\text{C}}$ . If the metal shim was made out of aluminum 6061, which has a thermal expansion of ( $\alpha = 23.6 \frac{\mu\text{m}}{\text{m}^\circ\text{C}}$ ), the thickness of this metal shim would have to be machined to about  $590 \mu\text{m}$ .

## Introduction

For a typical free-running External Cavity Diode Laser (ECDL), the frequency linewidth is about 1 MHz because of mechanical vibrations of the laser cavity, unavoidable fluctuations of the laser diode current source, and a slow temperature drift. To coherently manipulate the internal and external degrees of freedom of ultracold atoms and molecules, a laser with sub-MHz linewidth is required. The ultimate goal of my project is to build an ultrastable optical cavity that is used as a standard frequency reference to stabilize multiple ECDLs with sub-MHz linewidth. The stabilities of the ECDLs are determined by the stability of the optical cavity and bandwidth of the feedback loop.

The optical cavity is passively stabilized by precisely controlling its temperature. Furthermore, the material of the cavity support is ultralow expansion (ULE) glass with negligible thermal expansion coefficient ( $\alpha < 0.1 \times 10^{-6}/\text{K}$ ). However, other essential parts, such as cavity mirrors and a piezo transducer (PZT), have significant thermal expansion coefficients ( $\alpha > 1 \times 10^{-6}/\text{K}$ ). Even with a precise temperature control

( $\Delta T \sim 10 \text{ mK/day}$ ), the resonant frequency of the optical cavity would drift 40 kHz that could be significant for some narrow transitions. To reduce such possible thermal drift by a factor of ten, we plan to insert a shim metal piece between the ULE glass cavity support and the cavity mirror assembly. The entire system has a zero effective thermal expansion coefficient. Therefore, the thermal drift will be significantly reduced.

In this scheme, we need to know thermal expansion coefficients of all relevant parts, such as the cavity mirrors, the shimming metal, and the PZT with high precision. However, the vendor of the PZT can only provide a rough thermal expansion coefficient, as To reduce the effective thermal expansion coefficient of the entire system down to  $\alpha < 0.3 \times 10^{-6}/\text{K}$  ( $< 10 \text{ kHz/day}$  cavity resonance drift), we need an eight times more accurate measurement. To facilitate such accurate measurements, a rule with nanometer resolution is required. For example, if we increase the temperature of the PZT (2.5 mm thickness) by  $3^\circ\text{C}$ , the longitudinal dimension change is about 37.5 nm. In this work, we used a Michelson

interferometer to measure this material's change in length due to temperature.

### Background

The Michelson interferometer uses light interference to measure distances in units of wavelengths of light. It was developed by Albert Michelson in 1893, to measure the standard meter of the wavelength of the red line in the cadmium spectrum. The Michelson interferometer consists of two mirrors, mirror 1 and mirror 2, arranged as shown in figure 1. A collimated light beam from a laser is divided into two beams by a 50/50 beamsplitter. The reflected light beam (beam 1) by the beamsplitter is reflected by the mirror 1 and retraces the path to the same beamsplitter. The original transmitted light beam (beam 2) by the beam splitter is reflected by the mirror 2, retraces the path to the beamsplitter. The partially reflected light of the beam 2 is superimposed to the partially transmitted light of the beam 1 and is detected by a biased photodetector. The detected light intensity is determined by the phase difference of the light field in the beam 1 and the beam 2. Furthermore, the phase difference is proportional to the distance difference ( $d_1 - d_2$ ). Therefore, by measuring the intensity modulation, we can deduce the relative distance change between the two arms of the Michelson interferometer.

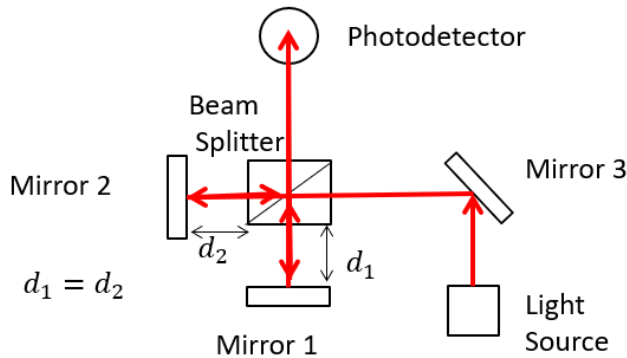


Figure 1 Balanced Michelson Interferometer

Next, let us quantitatively derive the above-mentioned intensity-distance relationship. The incident light beam is split to  $E_1 e^{ikx_1}$  and  $E_2 e^{ikx_2}$ , where  $E_1$  and  $E_2$  are the magnitudes of the electric fields of the beam 1 and the beam 2,  $k$  is the wavevector, and  $d_1$  and  $d_2$  represent different paths to the mirror 1 and the mirror 2. After the beam 1 and the beam 2 are combined, the total electric field of the light beam is  $E_R = E_1 e^{i(k2d_1)} + E_2 e^{i(k2d_2)} = E_1 e^{i\phi_1} + E_2 e^{i\phi_2}$ . The signal of the photodetector is proportional to the square of the electric field,  $I \propto |E|^2 = |E_1|^2 + |E_2|^2 + 2E_1 E_2 \cos(\phi_1 - \phi_2)$ . The first two terms are constant, while the last term is oscillating with the phase difference. If the beam splitter is balanced (50/50),  $E_1 = E_2$ . Thus, the detected signal  $I \propto 1 + \cos(\phi_1 - \phi_2)$ . When  $\phi_1 - \phi_2 = 2n\pi$ , where  $n$  is integer, the two beams are constructive interference, and the detected light intensity is maximum. When  $\phi_1 - \phi_2 = 2(n + 1)\pi$ , the two beams are destructive interference, and the detected light intensity is minimum. If we vary the phase difference continuously, we should be able to record an oscillating signal of the photodetector, named interferogram. The phase difference is determined by the distance difference of the two arms,  $\phi_1 - \phi_2 = 2k(d_1 - d_2)$ . In all, the relationship between the interferogram and distance change is:

$$I \propto \frac{E_0^2}{2} (1 + \cos(2k(d_1 - d_2))) \quad (1)$$

### Experimental Design

In our experiment, we implement a Michelson interferometer to measure a small spatial shift that is induced by the PZT thermal expansion. To suppress other shifts, the Michelson interferometer is balanced, as shown in Figure 1 and 2. The only difference in the two arms is: one arm has a PZT

between the mirror and the aluminum heated base, but the other arm has an aluminum shim that has the same geometry as the PZT. To control the temperature of the PZT, two resistive heaters are mounted symmetrically on a large aluminum plate, as shown in Figure 1 and figure 2. A CW diode laser was used and was measured to have a wavelength of 666nm.

Mount 1	mm	Mount 2	mm
Al Plate 1	3.15	Al Plate 2	3.15
Glass blank 1	12.17	Glass blank 2	12.17
AL Shim	2.50	PZT	2.50
Mirror 1	3.12	Mirror 2	3.12

Figure 2 Lengths of Mount Material

A thermocouple temperature sensor probes the temperature of the large aluminum plate. A COMSOL simulation shows the temperature inhomogeneity of the entire aluminum plate is smaller than 1°C. Because of the poor heat conduction of the PZT, the inhomogeneity of the PZT is as large as 2°C in the longitudinal direction. Such temperature inhomogeneity can be precisely determined by the COMOSL simulation and have been implemented in the data analysis. The other side of the large aluminum base plates are attached to a ULE glass blank with small thermal conductivity is. The ULE glass bank isolates the most of heat to the adjust mirror mounts that could induce imbalanced systematics.

The interferometer was then placed into an acrylic box. A function generator was used to create a ramping signal that was amplifier by a PZT driver that controlled the PZT. An oscilloscope was used to read the ramping signal, the amplified signal, the trigger signal, and last the interference signal of the interferometer.

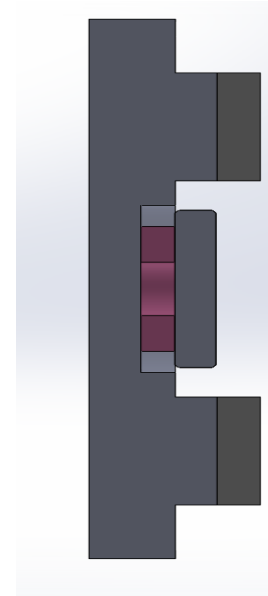


Figure 3 Design of Mount

### Procedure

The heaters for the experimental setup was left overnight so the system could reach thermal stability. The laser, function generator, driver, digital thermocouple reader, and oscilloscope were turned on. Once the interference signal was seen and the initial temperature was recorded, the data from the oscilloscope was saved onto a flash drive. The aluminum heating plates were then heated by increasing the voltage in the power supply to the power resistors. As the temperature increased the interference signal was then recorded in one-degree increments until three degrees above the initial temperature was reached. After the last sample was taken, the power supply's volage was then lowered. The interferometer was then left alone for about 10 minutes to cool down. his was then repeated multiple times.

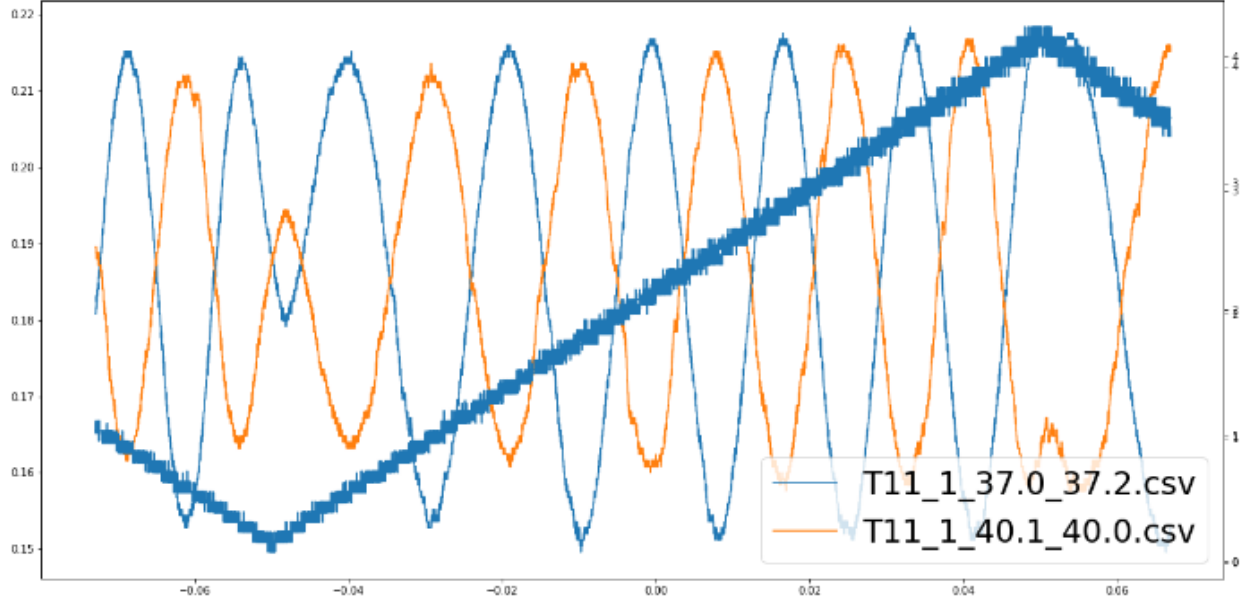


Figure 4 Interference Patters of Michelson Interferometer

## Data and Analysis

As stated in the previous section. It was decided to use a balanced Michelson interferometer. Using the previous equation, some factors were added to the equation to the variable that where changed.

$$E_1 = E_1^{(0)} e^{2ik(d_1 + \Delta L_{AL})}, \quad \text{where} \\ 2k(d_1 + \Delta L_{AL}) = \phi_1$$

$$E_2 = E_2^{(0)} e^{2ik(d_2 + \Delta L_m + \Delta L_{PZT})}, \quad \text{where} \\ 2k(d_2 + \Delta L_m + \Delta L_{PZT}) = \phi_2$$

Here,  $d_1$  and  $d_2$  are the distances of the mirrors from the beam splitter,  $\Delta L_{AL}$  is the change in length of the aluminum shim due to the change in temperature,  $\Delta L_{PZT}$  is the change in length of the PZT due to the change in temperature, and  $\Delta L_m$  is the change in distance due to the modulation of the PZT. The interference equation for the interference wave is stated here.

$$E_{interference} = E_1 + E_2 \\ = E^{(0)}(e^{i\phi_1} + e^{i\phi_2}) \quad (2)$$

Since the intensity is being measured,  $I \propto G[E^{(0)}]^2 (e^{-i\phi_1} + e^{-i\phi_2})(e^{i\phi_1} + e^{i\phi_2})$  and simplifying this equation,

$$I \propto G[E^{(0)}]^2 (1 + \cos(\phi_1 - \phi_2)).$$

This cosine term determines the interference of the resulting wave form. Since it is assumed that interferometer is balanced,  $d_1 - d_2$  is constant.  $\Delta L_m$  is the function of time and  $\Delta L_{PZT} + \Delta L_{AL}$  terms are functions of temperature.

$$\phi_1 - \phi_2 = \\ ((d_1 - d_2) + \Delta L_m + (\Delta L_{PZT} - \Delta L_{AL}))$$

(3)

Due to the balanced Michelson interferometer,  $d_1 - d_2$  is zero. Both the  $\Delta L_{PZT}$  and the  $\Delta L_{AL}$  are temperature dependent. The equation for thermal expansion can be seen here,  $\Delta L = L_0 \alpha \Delta T$ , where  $\Delta L$  is the change in length of the material in question due to the change in temperature,  $L_0$  is the original length of the material before heat is,  $\Delta T$  is the change in temperature of the material, and finally  $\alpha$  is the coefficient of thermal expansion of the material. This  $\Delta L_m$  term is time dependent

and determined by the PZT's displacement and voltage curve, which has a linear relationship of  $\frac{2\mu m}{100V}$ . A linear ramping curve of  $\frac{80V}{200ms}$  was used to drive the PZT, thus the displacement ramping voltage rate was calculated to be  $\frac{8\mu m}{s}$ . Therefore, by comparing the inference waves at different temperature, this  $\Delta L_m$  becomes a constant and this equation becomes:

$$\phi_{inference} = (2k((\alpha_{PZT}(T_1 - T_2) - \alpha_{AL}(T_1 - T_2)))) \quad (4)$$

The phase shift was determined from finding the time difference of the peaks on the time versus intensity data obtained from the oscilloscope. Differences between the peaks in the time domain was determined with peak detection using Python and verified by hand for four peaks at each temperature signal in the time domain. The time differences were then compared to the average period of the signal to determine the phase change in distance as a function of temperature increase of the system by this relation:  $\Delta L_{Total} = \frac{\lambda}{2} (\frac{\Delta\phi_{avg}}{2\pi})$ . A least squares linear regression was used to fit these data to determine  $\frac{\Delta L}{\Delta T}$ . The linear thermal expansion of the system is  $\Delta L_{Total} = L_{AL}\alpha_{AL}\Delta T_{AL} - L_{PZT}\alpha_{PZT}\Delta T_{PZT}$ .

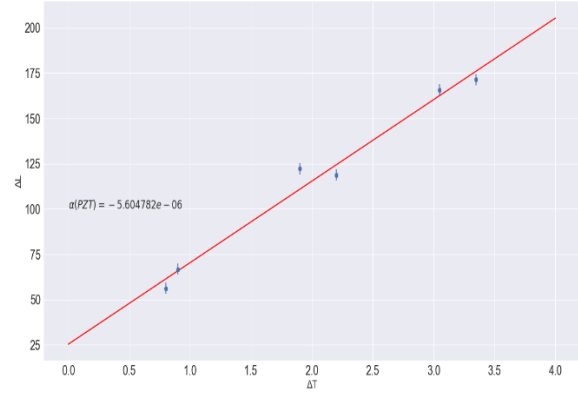


Figure 5 Linear Regression of the Data

### Conclusion

This leads to  $\alpha_{PZT} = \frac{\Delta L_{Total} - L_{AL}\alpha_{AL}}{L_{PZT}}$  if it is assumed  $\Delta T_{AL} \cong \Delta T_{PZT}$ . The coefficient of thermal expansion of the PZT was found to be  $-5.6 \pm 1.2 \frac{\mu m}{m^{\circ}C}$ . To offset this thermal drift, the metal shim will have to match the thermal drift of the PZT. If we increase the temperature of the PZT (2.5 mm thickness) by 3 °C, the longitudinal dimension change is about 42 nm when the newfound coefficient of thermal expansion. If the metal shim was made out of aluminum 6061, which has a thermal expansion of  $\alpha = 23.6 \frac{\mu m}{m^{\circ}C}$ , the thickness of this metal shim would have to be machined to about 590  $\mu m$ . To reduce the systematic error, the more accurate temperature control in need to maintain the temperature of the of the arm.